1.0

1.1

1.25   1.4

2.5

2.2

2.0

1.8

1.6

physical dynamics, inc.

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Final Report   January 1982

The Excitation of Atmospheric
Gravity Waves By
Unstable Shears

by

David C. Fritts
Physical Dynamics, Inc.
Bellevue, WA 98009

Prepared for

Air Force Office of Scientific Research
Bolling AFB, DC 20332

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Research performed under this AFOSR contract addressed two aspects of gravity wave excitation by unstable shear layers in the atmosphere; the non-linear excitation of radiating gravity waves and the response of the mean velocity profile to gravity wave growth and decay. Gravity excitation via the nonlinear interaction of two smaller-scale Kelvin-Helmholtz (KH) instabili-ties was found to be significantly faster than that predicted by linear

stability analysis, accounting for the geophysical significance of such waves. Gravity waves excited in this manner were observed to achieve large amplitudes and to be very localized during excitation.

Mean velocity profiles were found to respond in several ways to evolving, unstable gravity waves. Radiating waves excited by the nonlinear interaction of two KH modes were observed to produce relatively small mean flow accelerations away from the shear layer because of source shear stabilization by the KH modes. Large mean flow accelerations were induced by unstable modes evolving in isolation. These results suggest that gravity waves generated by unstable shears may be important in the transport of energy and momentum throughout the atmosphere.

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I. STATEMENT OF WORK

The following research tasks were performed under this AFOSR contract:

1. Investigation of the nonlinear excitation of internal gravity-wave modes at an unstable shear via the vortex-pairing mechanism in order to understand

    a) the manner in which excitation occurs;

    b) why certain modes are excited while others may be suppressed;

    c) the influence of other factors such as dissipation and propagating-versus-evanescent behavior on the efficiency of excitation; and

    d) whether sustained radiation may occur.

2. Examination of the transport and redistribution of momentum by unstable modes to assess

    a) which modes (evanescent or vertically-propagating) can most effectively extract momentum from and stabilize an unstable velocity shear; and

    b) what effects may be induced in the atmosphere by the redistribution of momentum by unstable radiating modes.

II.   STATUS OF RESEARCH EFFORT

A.   Background

Gravity waves play an important role in a variety of atmospheric processes. Their effects include, among others, the production or modulation of turbulence through wave-induced velocity and bouyancy fluctuations (Hines, 1963; Orlanski and Bryan, 1969), the initiation and organization of convection (Uccellini, 1975), the excitation of motions on other scales through nonlinear interactions (Bretherton, 1969a; Fritts, 1979), and perhaps most important, the transport and redistribution of energy and momentum throughout the atmosphere (Eliassen and Palm, 1961). Momentum deposition by gravity waves, for example, is now thought to account for the enormous mean flow accelerations observed in the mesosphere (Lindzen, 1981).

Gravity waves, likewise, are excited in many ways. Important sources include orographic forcing (Bretherton, 1969b; Klemp and Lilly, 1975), convective activity, and frontal accelerations. Perhaps the dominant source of atmospheric gravity waves, however, is wind shear (Einaudi, Lalas, and Perona, 1978/79). Thus, it is important to understand the detailed mechanism(s) by which shear excitation occurs.

A recognition of the importance of wind shear in the excitation of atmospheric gravity waves has motivated numerous investigations of the subject. Several studies found a Helmholtz velocity profile to support neutral internal gravity waves in addition to the familiar Kelvin-Helmholtz (KH) instability (Drazin and Howard, 1966; Lindzen, 1974). Other studies found a lower boundary to destabilize an unstable velocity shear to long wavelength, vertically propagating disturbances (Jones, 1968; Lindzen and Rosenthal, 1976; Lalas and Einaudi, 1976). In these studies, however,

linear stability analysis found the growth rates of vertically propagating modes to be considerably less than that of the most rapidly growing KH mode. It is difficult, therefore, to explain the observed importance of such shear-excited gravity waves with linear theory alone.

Results of a nonlinear investigation of the shear excitation of atmospheric gravity waves are pesented in this report. These results demonstrate that the excitation of a vertically propagating gravity wave via the nonlinear interaction of two KH instabilities can be rapid and energetic, providing a viable explanation for the occurrance of such waves in the atmosphere.

B.   Research Results

Research performed under this contract focussed on the role of nonlinearity in both the excitation of vertically propagating gravity waves and the mean flow modifications induced by these waves. The specific research objectives that guided this study were presented in the previous section. The significant findings of this research are discussed here. A more quantitative discussion of these results is presented in the manuscript included as an appendix to this report.

The most obvious shortcoming of linear stability theory is its inability to predict radiating gravity wave modes that have growth rates competitive with that of the KH instability for general, unstable atmospheric velocity profiles. Nonlinear simulations performed under this contract have shown, however, that the interaction of two KH instabilities growing in unison can excite, very efficiently, a large-amplitude radiating wave, as predicted by Chimonas and Grant (1982).

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The character of the nonlinear radiating wave excitation differs in several important ways from that predicted by linear stability theory. Whereas a radiating wave growing at its (small) linear growth rate has sufficient time to achieve a global structure (in which the wave perturbation occupies a large vertical domain), a radiating wave excited in a nonlinear fashion simply grows too quickly to achieve a global structure during excitation. This is because a radiating wave can only transport information (or energy, momentum, structure, etc.) at its vertical group velocity, which is independent of the growth rate. In a nonlinear excitation, a radiating wave grows at a rate equal to the sum of the growth rates of the two KH modes providing the forcing. Thus its growth rate far exceeds any growth rate predicted by linear theory.

The nonlinear growth mechanism proposed by Chimonas and Grant (1982) is essentially an off-resonant triad interaction involving two KH modes with large growth rates and an unstable gravity wave mode with a small, linear growth rate. However, the simulations conducted under this contract indicate that the requirement for nonlinear excitation is even less restrictive. Whereas resonant interactions in the presence of a lower boundary do appear to excite rapidly a disturbance that closely resembles a radiating mode with a small linear growth rate, similar radiating wave excitation occurs for a shear layer in an unbounded fluid for which no unstable gravity wave modes exist. Thus, the radiating wave component of the resonant triad need not be unstable to be excited in a nonlinear fashion.

This research addressed not only the nature of nonlinear radiating wave excitation, but also the selection of radiating wave modes and the efficiency of excitation. Results of these studies suggest that these factors are governed entirely by the KH modes interacting to excite the radiating wave. As mentioned earlier, the

insensitivity of the KH modes to the available, unstable radiating modes indicates that the nonlinear forcing cannot be governed by the vertical structure of these modes. Instead, the radiating wave that emerges has a vertical structure determined by the phase velocities of the forcing KH modes. The structure of the emerging wave simply resembles that of the unstable gravity wave mode (if one exists) that best matches the phase velocities and envelope wavelength determined by the two KH modes. Thus, for symmetric profiles of shear and stratification, the most likely nonlinearly-excited gravity wave will have similar structure above and below the source shear. Other possible wave modes with different structure (evanescent vs. propagating) above and below the source shear have horizontal phase velocities that cannot couple efficiently with the forcing KH modes. The effects of dissipation, while influencing the growth rates of the KH modes (and the forced radiating wave) to some degree, appear to have a much larger impact on the linear and quasi-linear (including mean flow modifications) growth of the radiating wave modes. This supports the conclusions of the linear, viscous stability analysis conducted by Davis and Peltier (1977) as well.

One concept that has received some attention in the literature is that of sustained radiation of gravity wave energy from an unstable shear layer. Lindzen (1974) proposed that such radiation could tend to extract energy from, and therefore stabilize, an unstable shear. Other researchers, however, showed that neutral gravity waves could not stabilize a shear layer in that manner (Acheson, 1976; Grimshaw, 1976; McIntyre and Weissman, 1978). Quasi-linear simulations of radiating wave growth conducted as a part of this research have helped resolve this issue. In the event that such radiating wave excitation (in the absence of KH modes) is possible, as can occur for special types of shear and stratification profiles, the actual situation is somewhere between that invisioned by Lindzen and the other researchers. In practice,

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no finite amplitude wave motion can be established without effecting mean flow changes. This is because the mean velocity profile undergoes an acceleration as a consequence of a divergence in the gravity wave Reynolds stress, and no gravity wave can be established without producing a Reynolds stress divergence. Thus as an unstable gravity wave grows toward large amplitude, it experiences a Reynolds stress divergence at the source level (again a consequence of the finite vertical group velocity) which causes the mean shear at that level to decrease. The decreasing shear results in a reduction in the growth rate of the unstable mode, ultimately resulting in a state of sustained radiation (in the absence of dissipation). Once a state of sustained radiation has been achieved, the wave ceases to cause stabilization of the mean velocity shear.

The evolution toward a state of sustained radiation is similar in a dissipative medium. In this case, however, no steady state is achieved because of continual viscous erosion of the mean velocity profile, causing the wave motion to undergo a gradual decay. Nevertheless, this state of quasi-sustained radiation can lead to substantial transport of mean momentum through the source shear from one region where the wave is transient or dissipating to another. This process may play an important role in the transport and redistribution of energy and momentum by gravity waves throughout the atmosphere. Additional details are given in the appendix.

The quasi-sustained radiation discussed above is only one type of radiation associated with propagating modes at an unstable shear layer. Other modes, in addition to those that have propagating character both above and below the shear layer, are also possible. These other modes, whose structure is described by Fritts (1980), evolve very differently from either the KH mode or those modes that can achieve a state of quasi-sustained radiation. Modes that cannot attain a state of

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quasi-sustained radiation are observed to have propagating character either above or below the source shear. Elsewhere, they are vertically evanescent. This means that even with small or zero growth, such modes have a Reynolds stress divergence, and therefore a continuing mean flow acceleration, at or near the source shear. The effect of such waves on the mean flow, then, is to transport mean momentum to, not through, the unstable shear layer from regions where the wave is transient or dissipating. This precludes attainment of a state of quasi-sustained radiation where mean flow changes at the source shear are a consequence only of dissipation. Unstable mode evanescence above and/or below a source shear, then, results in a wave disturbance with a much shorter lifetime than a mode of similar growth rate with propagating character above and below the shear.

Several effects of gravity wave propagation were mentioned at the beginning of this section. Perhaps the most important of these for shear-excited gravity waves are the generation of turbulence and the transport of energy and momentum. As observed in both quasi-linear and nonlinear simulations, unstable mode evolutions result in significant mean flow accelerations both at the source shear and in regions where the radiating waves are transient or dissipating. For gravity waves in the atmosphere, the dominant dissipation mechanism is likely to be wave breaking, which results in the production of turbulence and allows the wave-induced mean flow changes to become permanent (momentum deposition). Various unstable modes may also cause the formation of velocity jets below the source shear. Such jets are a result of the modal structure and the transport of mean momentum through the source shear by the wave, and may themselves contribute to the production of turbulence and the creation of banded structures observed in the planetary boundary layer (Gossard, Richter, & Atlas, 1970).

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III. PRESENTATIONS, PUBLICATIONS, AND INTERACTIONS

This research effort resulted in one presentation at an international meeting and the submission of two papers, prepared all or in part with AFOSR support, to scientific journals. In addition, this research facilitated numerous interactions between the Principal Investigator and other researchers in the atmospheric sciences community.

A paper entitled "A mechanism of gravity wave excitation observable with atmospheric radars" was presented at the 20th AMS Conference on Radar Meteorology held in Boston in December 1981. A written version of the paper appeared in the conference preprint volume.

Two manuscripts were prepared and submitted for journal publication during the contract year. The first, entitled "Shear excitation of atmospheric gravity waves," presents results of the AFOSR-sponsored research and was submitted to J. Atmos. Sci. This paper is included as an appendix to this report. The second manuscript entitled "The transient critical-level interaction in a Boussinesq fluid," was prepared for publication with partial AFOSR support and submitted to J. Geophys. Res. A third paper, which will extend the results of the first paper to unbounded flows and realistic atmospheric profiles, is planned. It is anticipated that this paper will also be submitted to J. Geophys. Res.

This research benefitted from numerous interactions of the Principal Investigator with scientists at the University of Washington, where the PI presented a seminar, the National Center for Atmospheric Research, where most of the numerical simulations were performed, the Aeronomy Laboratory of the National Oceanic and Atmospheric Administration, and other institutions.

Expertise gained in performing this research has contributed to requested peer reviews by the Principal Investigator of three scientific contributions:

- a scientific proposal for atmospheric observations using a bistatic radar, requested by Dr. Ronald C. Taylor of the National Science Foundation;

- one paper submitted to J. Atmos. Sci.; and

- one paper submitted to Geophys. Res. Ltrs.

IV. PERSONNEL

This research was performed by Dr. David C. Fritts as a staff scientist at Physical Dynamics Northwest. A professional biography of Dr. Fritts follows.

DAVID C. FRITTS
STAFF SCIENTIST
PHYSICAL DYNAMICS, NORTHWEST
P.O. BOX 3027
BELLEVUE, WASHINGTON 98009

Specialized professional competence

- Atmospheric physics
- Gravity wave propagation
- Stability analysis

Representative recent research

- Development of analytic and numerical models of the gravity wave-mean flow interaction
- Investigations of nonlinear gravity wave-critical level interactions
- Nonlinear studies of unstable wave excitation and growth
- Stability analysis of unstable velocity profiles
- Development of stability criteria
- Oceanic internal wave studies

Other professional experience

- Teaching and research assistant in physics, University of Illinois, 1971-1977
- Postdoctoral research, National Center for Atmospheric Research, 1977-1978
- Postdoctoral research, Aeronomy Lab, National Oceanic and Atmospheric Administration, 1978-1979
- Staff scientist at Physical Dynamics Northwest, 1979-1981

Academic background

- B.A. in physics (1971), Carleton College
- M.S. in physics (1973), University of Illinois
- Ph.D. in physics (1977), University of Illinois

Publications and presentations

- Five publications on gravity wave-mean flow interactions
- Two papers submitted for publication
- Presentations at international meetings

Professional associations

- American Meteorological Society
- American Physical Society

V. CONCLUSIONS

The results discussed in this report (and in the appendix) demonstrate that nonlinearity plays an important and perhaps dominant role in both the excitation of vertically propagating gravity waves by unstable shears and the mean flow modifications induced by shear excited gravity waves. Nonlinear simulations have shown that the interaction of two KH modes of similar but unequal wavelength can excite very rapidly a large-amplitude, vertically propagating gravity wave, providing a feasible explanation for the occurrance of such waves in the atmosphere. The consequence of rapid nonlinear excitation is a gravity wave packet that is transient and vertically localized, as distinct from the slowly growing, global disturbances predicted by linear stability theory. This excitation mechanism appears to be relatively robust and may, therefore, contribute to the maintenance of the internal wave spectrum throughout the atmosphere.

Gravity waves generated at unstable shear layers, either in isolation or through the nonlinear interaction of two KH modes, were observed to induce significant accelerations of the mean state, both near and at large distances from the source shear. In all simulations, the shear layer was found to be stabilized by the growing disturbances. Mean flow modifications away from the source shear were found to depend upon a number of factors, including the wave amplitude, the duration of excitation, and the wave structure. Unstable modes that evolved toward large amplitude slowly produced the largest mean flow accelerations far from the shear layer. Rapidly excited gravity waves resulted in faster but smaller accelerations of the mean flow. Because of their relative importance, therefore, such waves may be expected to contribute significantly to the transport of energy and momentum throughout the atmosphere.

APPENDIX:

Shear Excitation of Atmospheric
Gravity Waves

by

David C. Fritts

ABSTRACT

Unstable velocity shears are a common source of vertically propagating gravity waves in the atmosphere. However, the growth rates of unstable modes predicted by linear theory cannot account for their observed importance.

We examine in this paper, using a numerical model, the nonlinear excitation and evolution of atmospheric gravity waves. It is shown that such waves can reach large amplitudes and induce significant accelerations of the mean velocity profile, resulting in shear stabilization and jet formation where the gravity wave is dissipated. Unstable modes that are vertically propagating above and below the shear layer may, when growing in isolation, achieve a state of quasi-sustained radiation.

The nonlinear excitation of vertically propagating gravity waves via the interaction of two KH modes is found to be very rapid, providing an explanation for their occurrance in the atmosphere.

1. INTRODUCTION

The excitation of gravity waves by unstable shear layers is the most common (Einaudi, Lalas, and Perona, 1978/79) but perhaps least understood mechanism of gravity wave excitation in the lower atmosphere. This is because this excitation mechanism, unlike other important sources such as orographic forcing, convective activity, and frontal accelerations, appears to be an inherently nonlinear process. In this paper, we examine the shear excitation of atmospheric gravity waves and illustrate the role of nonlinearity in gravity wave growth and the accompanying mean flow evolution.

The importance of gravity waves in the atmosphere, and therefore the shear excitation mechanism, derives from their ubiquitous nature and their many atmospheric effects. Because gravity waves may propagate to considerable heights, they play an important role in a variety of atmospheric processes. Their effects include, among others, the transport and deposition of momentum and energy, resulting in significant mean flow accelerations (Holten and Lindzen, 1972; Lindzen, 1981; Dunkerton, 1981), the initiation and organization of convective activity (Uccellini, 1975; Einaudi and Lalas, 1975), and the generation and modulation of turbulence (Hodges, 1967; Bretherton, 1969). These and other effects have motivated numerous studies of gravity wave excitation by unstable shear layers.

A number of authors have observed that piecewise continuous or discontinuous velocity profiles in an unbounded, stratified fluid can support neutral internal gravity waves in addition to the well-known Kelvin-Helmholtz (KH) instability (Jones, 1968; Lindzen, 1974; Acheson, 1976; Grimshaw, 1976; McIntyre and Weissman, 1978). Several studies have demonstrated that the presence of a rigid surface below an

unstable shear layer can destabilize the shear to long wavelength disturbances. Jones (1968) found a piecewise-continuous velocity profile to be unstable to all wavelengths, and Lalas, Einaudi, and Fua (1976) and Lindzen and Rosenthal (1976) found a Helmholtz velocity profile with a rigid lower boundary to possess an infinite number of long wavelength modes. Investigations of continuous velocity profiles found these to permit a finite and predictable number of unstable modes (Lalas and Einaudi, 1976; Davis and Peltier, 1976; Fritts, 1980).

At first glance, the linear studies cited above appear to explain the occurrance of long wavelength, vertically propagating gravity waves with unstable velocity shears. There is, however, a serious deficiency in this linear explanation; the growth rates of the long wavelength modes predicted by linear theory are always significantly less than the growth rate of the KH mode. According to linear theory, then, the KH mode should always dominate the breakdown of an unstable shear layer, and longer wavelength modes should never be observed. That long wavelength vertically propagating modes are observed to be excited by unstable shear layers suggests that the excitation mechanism is nonlinear.

At least two previous studies have addressed the nonlinear excitation of vertically propagating gravity waves by an unstable shear layer. The study by Davis and Peltier (1979) proposed that radiating wave excitation occurs via the vortex pairing (and wavelength doubling) of the most rapidly growing KH mode. These authors used a weakly nonlinear theory to test their hypothesis. A second study by Chimonas and Grant (1981) argued that the nonlinear interaction of two KH modes of similar wavelength was a more efficient means of exciting a long wavelength, radiating gravity wave. In this paper, we adopt the view of Chimonas and Grant and examine the nonlinear interaction of two KH modes of similar wavelength.

The purpose of this paper is to illustrate both the mechanism and the conse-quences of excitation of a vertically propagating gravity wave by an unstable velocity shear. We begin by reviewing the results of the linear stability analysis of Lalas and Einaudi (1976) and Fritts (1980) in Section 2. Section 3 provides a discussion of the numerical model used for the quasi-linear and nonlinear simulations of radiating wave excitation. Results of the linear stability analysis are needed for the quasi-linear study discussed in Section 4. These simulations are intended to illustrate the radiating wave growth, stabilization, and decay and the induced shear stabilization and jet formation (resulting from momentum transport by the wave) well away from the source shear. Results of the quasi-linear simulations suggest that a radiating gravity wave may cause substantial mean flow accelerations and, depending upon its character above and below the shear layer, achieve a state of quasi-sustained radiation, as anticipated by McIntyre and Weissman (1978). The nonlinear excitation of a radiating gravity wave via the interaction of two KH modes is examined in Section 5. This excitation is found to be very efficient, proceeding quickly and resulting in a transient gravity wave of large amplitude. The conclusions of this study are presented in Section 6.

2. LINEAR STABILITY ANALYSIS RESULTS

We summarize here results of the linear stability analysis conducted by Lalas and Einaudi (1976) and Fritts (1980). Results presented here were obtained with a constant Brunt-Vaisala frequency, $N_o(z) = \sqrt{g/H}$, and a velocity profile of the form

$$\bar{u}(z) = u_o \tanh \left[(z-z_i)/h \right] \; , \tag{1}$$

with $\sigma = h/H = 0.01$. The flow is assumed bounded by a rigid surface at $z=0$. While not representative of general atmospheric conditions, these profiles permit a very simple stability analysis and an assessment of the importance of gravity wave excitation and propagation in the real atmosphere. The significant results of this analysis are listed below:

1. There are, in general, a number of modes associated with an unstable shear sufficiently far from a rigid lower boundary. All but the KH mode are a consequence of the boundary.

2. The unstable modes comprise two families, each family containing modes with $n = 0, 1, 2 \ldots$ half wavelengths between the shear layer and the lower boundary such that

$$n \leq (z_i/h - 1)/\pi \; . \tag{2}$$

Thus, an unstable shear layer far from the earth's surface supports more unstable modes than the same shear layer near the ground. The unstable modes that occur for the above $N(z)$ and $\bar{u}(z)$ profiles with $z_i/h = 20$ are illustrated in Figure 1.

3.   The two mode families have different structures above the shear layer. One family (type A) contains modes, including the KH mode, that are vertically evanescent above the shear layer. The second family (type B) includes modes that are vertically propagating above the shear layer. The vertical structure of the type A and B modes is illustrated in Figure 2 for $z_j/h = 20$ and a minimum mean Richardson number, Ri = 0.10.

4.   For a given Ri, modes with less structure (fewer half wavelengths) below the shear layer have larger growth rates than other modes in the same family. For a given n, type A modes have larger growth rates than type B modes.

The character of the various unstable modes described above enables us to anticipate their atmospheric effects, assuming that they can be excited efficiently. The KH mode (A0 in Figures 1 and 2) has by far the largest linear growth rate of all the unstable modes, yet its direct atmospheric effects are limited to the immediate vicinity of the unstable shear because of its vertical structure. The KH mode can, however, excite through nonlinear interactions motions on larger horizontal scales which induce more global effects (see Section 5). Other type A modes, if they can be easily excited, can be expected to confine wave motions, induced mean motions, and other effects to the region near or below the shear layer. Type B modes, on the other hand, may have effects far above and below the source shear if they can be excited efficiently. These atmospheric effects are discussed in Section 4.

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3.   MODEL DISCUSSION

The model used for studies of gravity wave excitation and propagation solves the nonlinear, Boussinesq Navier-Stokes equations in two dimensions. It is assumed that the mean state is in hydrostatic balance and that the effects of the earth's rotation are negligible. Adopting a streamfunction-vorticity representation and choosing as scaling parameters a length scale, z_o, a velocity scale, u_o, a reference density, ρ_{oo}, and a density difference, ρ_o, the equations of motion take the form

$$n_t + J(\psi, n) + Ri_o \rho_x' - \frac{1}{Re} \nabla^2 n = 0, \tag{3}$$

$$\rho_t + J(\psi, \rho) - \frac{1}{RePr} \nabla^2 \rho = 0, \tag{4}$$

and

$$n = -\nabla^2 \psi, \tag{5}$$

where

$$Ri_o = \frac{g}{\rho_{oo}} \frac{\rho_o z_o}{u_o^2}, \tag{6}$$

$$Re = u_o z_o / \nu, \tag{7}$$

$$Pr = \nu / \kappa, \tag{8}$$

and $J(,)$ is the nondimensional Jacobian operator.

A 6

As in Fritts (1978), the streamfunction, vorticity, and density are written as truncated Fourier series in the horizontal coordinate,

$$
\left\{
\begin{array}{c}
\psi(x,z,t) \\
\eta(x,z,t) \\
\rho(x,z,t)
\end{array}
\right\}
= \sum_{n=-N}^{N}
\left\{
\begin{array}{c}
\psi^n(z,t) \\
\eta^n(z,t) \\
\rho^n(z,t)
\end{array}
\right\}
e^{i\alpha n x},
\tag{9}
$$

where α is the horizontal wavenumber of the incident wave. Substituting (9) into (3) – (5) and collecting coefficients of $e^{i\alpha n x}$, we obtain $6N + 3$ equations describing the evolution of the perturbation and mean state fields,

$$
\eta^n_t + \psi^n_x \eta^o_z - \eta^n_x \psi^o_z + \sum_{\substack{m=-N \\ |n-m| \le N, \, m \ne 0, n}}^{N} J(\psi^m, \eta^{n-m}) - \frac{1}{Re} \nabla^2 \eta^n + Ri_o \rho^n_x = 0,
\tag{10}
$$

$$
\rho^n_t + \psi^n_x \rho^o_z - \rho^n_x \psi^o_z + \sum_{\substack{m=-N \\ |n-m| \le N, \, m \ne 0, n}}^{N} J(\psi^m, \rho^{n-m}) - \frac{1}{RePr} \nabla^2 \rho^n = 0,
\tag{11}
$$

$$
\eta^n = -\nabla^2 \psi^n,
\tag{12}
$$

$$
\eta^o_t + \sum_{\substack{m=-N \\ m \ne 0}}^{N} J(\psi^m, \eta^{-m}) - \frac{1}{Re} \eta^o_{zz} = 0,
\tag{13}
$$

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$$\rho^o_t + \sum_{\substack{m=-N \\ m\neq 0}}^{N} J(\psi^m, \rho^{-m}) - \frac{1}{RePr}\rho^o_{zz} = 0, \qquad (14)$$

and

$$\eta^o = -\psi^o_{zz}. \qquad (15)$$

These equations are finite differenced in z and t, as described by Fritts (1979). A simulation is referred to as quasi-linear if only one mode is retained (N=1) in the above equations. Nonlinear simulations included several modes (N > 1).

All simulations were performed with a velocity profile given by (1) and a Brunt-Vaisala frequency squared, $N_o^2(z) = 0.001$ sec^{-2}. A rigid boundary was imposed at z=0, and a damping layer occupying the upper third of the model domain was used to simulate a radiation conditon for vertically propagating gravity waves (Fritts, 1979).

Unstable modes were excited in the model by introducing one or more small, localized, vertical velocity perturbations with specified horizontal wavelengths at the unstable shear layer at t=0. The unstable shear layer was allowed to select the dominant waveforms in the subsequent evolution.

4. QUASI-LINEAR EVOLUTION AND MEAN FLOW ACCELERATION

We examine in this section the evolution, in isolation, of two type B modes. A quasi-linear simulation of mode B1 shows that, after its initial growth, this mode attains a state of quasi-sustained radiation (gradual decay) in which mean momentum is transported through the shear layer, producing velocity jets both above and below the source shear. Mode B0, which is evanescent below the shear layer, is observed to stabilize the unstable shear by transporting mean momentum into the shear layer from the damping layer, where the wave is dissipated.

4.1 Mode B1 Evolution

The quasi-linear simulation of a mode B1 evolution was performed for a minimum mean Richardson number, Ri = 0.05, a shear layer depth, h = 100 m, a shear layer height, z_i = 2000 m, a model domain depth, d = 6000 m, a mean velocity, u_o = 14 m/sec, an eddy viscosity, $\nu = 0.14$ m^2/sec, and an eddy Prandtl number, Pr = 0.73. The eddy viscosity was chosen to be typical of values observed throughout the lower atmosphere (Gossard and Hooke, 1975). An initial vertical velocity perturbation of 0.001 m/sec with a horizontal wavelength λ_x = 2000 π m resulted in the excitation of mode B1 (see Figures 1 & 2). The simulation was carried out to 50 wave periods ($\sim$ 6 hrs.) measured relative to the outer flow.

The gravity wave structure is illustrated in Figure 3 at 40 wave periods with the perturbation streamfunction and total density fields. The horizontal and vertical perturbation velocities of the gravity wave at 40 wave periods at the edge of the model domain are shown in Figure 4. There is no phase shift with height below the shear layer because the wave is totally reflected at the lower boundary. This

reflection, together with wave over-reflection at the unstable shear layer, provides the mechanism for unstable mode growth (Lindzen and Rosenthal, 1976). Above the shear layer, the gravity wave structure exhibits a phase shift with height that is characteristic of vertical propagation. The gravity wave Reynolds stress, $-\overline{uw}$, is positive here, implying an upward transport of negative (or leftward) momentum. This momentum flux induces Eulerian mean flow accelerations wherever the gravity wave is transient or dissipating.

Illustrated in Figures 5 and 6, respectively, are the evolutions of minus the gravity wave Reynolds stress and the Eulerian mean velocity profile. Both are shown at 20, 30, 40 and 50 wave periods. Until 20 wave periods, wave amplitudes are small, mean flow modifications are slight, and the wave grows at nearly its linear growth rate. As the wave continues to grow, however, the vertical Reynolds stress divergence becomes large, particularly at the level at which the wave is being excited. The Reynolds stress divergence induces an Eulerian mean flow acceleration of the form (Andrews and McIntyre, 1976)

$$\overline{u}_t = - \frac{\partial}{\partial z} (\overline{uw}).$$ (16)

Thus, the gravity wave grows by extracting excess energy from the unstable shear, and in so doing, decreases the shear. In addition, the Eulerian mean flow experiences an acceleration below and a deceleration above the shear layer where the Reynolds stress divergence is also nonzero.

The decreasing mean velocity shear at the level of wave excitation causes a decrease in the growth rate of the unstable mode and a reduction in the Reynolds stress divergence at this level (Figure 5b). Ultimately, the shear layer is no longer

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unstable to this wavelength disturbance (Figure 5c), and additional erosion of the velocity shear by eddy viscosity and diffusivity results in gradual wave decay (Figure 5d). At this stage of the evolution, the Reynolds stress divergence at the level of wave excitation is nearly zero, a consequence of mode B1 propagation both above and below the shear layer. Thus, the wave ceases to modify the Eulerian mean flow at the shear layer. Instead, the regions of large divergence (and continued mean flow acceleration) are now below and well above the shear layer, where the wave is dissipated. The result is a transfer by the wave of mean flow momentum from upper levels of the flow through the shear layer to the region near the lower boundary where a velocity jet is produced (Figure 6 b to d).

In the case of negligible dissipation, the point at which the growth rate equals zero would correspond to a state of sustained radiation, as suggested by McIntyre and Weissman (1978), provided that the modal structure is relatively insensitive to continuing Eulerian mean flow modifications away from the shear layer. Sustained radiation appears possible, however, only for modes that are vertically propagating both above and below the shear layer. The shear stabilization which results is a consequence of the wave excitation [see equation (16)] rather than propagation of the wave away from the source shear, as proposed by Lindzen (1974).

4.2   Mode B0 Evolution

A mode B0 simulation was performed with most of the same wave and mean flow parameters used in Section 4.1. In order to excite a B type mode that was vertically evanescent below the shear layer, however, a shear layer height, z_i = 500 m, and an initial velocity perturbation of 0.0001 m/sec at a horizontal wavelength, λ_x = 2000 π m, were used. An eddy viscosity of 0.28 m^2/sec was chosen to retard slightly the more rapid growth and decay of the B0 mode. The simulation was carried out to 16 wave periods ($\sim$ 1½ hrs.).

The B0 mode structure at 16 wave periods is shown in the perturbation streamfunction and total density fields in Figure 7. The corresponding horizontal and vertical velocity perturbations at 16 wave periods are illustrated in Figure 8. Notice that mode B0 rapidly achieves a large maximum amplitude, but has a more transient nature than mode B1 (see Figure 3). This is a consequence of the rapid growth of, and more efficient shear stabilization by, mode B0, owing to its large Reynolds stress divergence within the shear layer. Below the shear layer, mode B0 is vertically evanescent and has no phase shift with height. Above the shear layer, however, mode B0, like mode B1, has a positive Reynolds stress, $-\overline{uw}$, implying an upward transport of negative momentum from the shear layer vicinity to the region where the wave is dissipated.

The evolutions of minus the gravity wave Reynolds stress and the Eulerian mean velocity profile are illustrated in Figures 9 and 10, respectively, at 10, 12, 14, and 16 wave periods. As in the B1 mode evolution, the wave growth is linear at early times. As mode B0 grows, its Reynolds stress divergence at the unstable shear lessens the shear and reduces the B0 mode growth rate (see Figures 9 and 10, a and b). Unlike mode B1, however, mode B0 does not cease shear stabilization as its growth slows and reverses. Instead, mode B0 continues to deposit positive (or rightward) momentum at or near the shear layer, accelerating its decay. This continued deposition of mean momentum at the source shear prevents mode B0 from achieving a state of quasi-sustained radiation, as observed in Section 4.1. It further precludes the large, quasi-linear Eulerian mean flow accelerations accompanying quasi-sustained radiation.

The quasi-linear B0 and B1 mode evolutions discussed here suggest that modes that are vertically propagating both above and below an unstable shear layer are more likely to play an important role in atmospheric dynamics, because of their less

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transient nature and larger mean flow accelerations, than modes that are vertically evanescent below (or above) the shear layer. The extent to which these quasi-linear evolutions represent the shear excitation of gravity waves in the atmosphere depends on the actual role of nonlinearity in the excitation of such modes. It will be shown in the following section that the nonlinear interaction of two KH modes can excite very rapidly a mode B1-like disturbance that is very transient in nature. But, however these modes are excited, their effects on the mean flow remain the same. Shear stabilization and Eulerian mean flow accelerations well away from the unstable shear are inescapable consequences of the excitation of vertically propagating gravity waves.

5. NONLINEAR EXCITATION

A nonlinear (N=5) simulation was performed to illustrate the excitation of a vertically propagating gravity wave via the interaction of two KH modes. The simulation used a minimum mean Richardson number, $Ri = 0.05$, a mean flow velocity $u_o = 14$ m/sec, a shear layer depth, $h = 100$ m, and a domain depth, $d = 6000$ m. As in the quasi-linear mode B1 simulation, a shear layer height, $z_i = 2000$ m, was chosen. Initial vertical velocity perturbations of 0.0001 and 0.0002 m/sec were specified at the unstable shear at horizontal wavelengths of 400π and 500π m, respectively. These perturbations resulted in KH modes which interacted to excite a vertically propagating gravity wave of horizontal wavelength 2000π m. An initial eddy viscosity of 0.14 m^2/sec was assun 1. In order to simulate the effects of nonlinear breakdown and turbulent dissipation, the eddy viscosity acting on the KH modes was increased to 14 m^2/sec at 1.0 wave periods, to 56 m^2/sec at 1.4 wave periods, and to 140 m^2/sec at 1.6 wave periods. These eddy viscosities are consistent with the turbulent values observed by Kennedy and Shapiro (1980). The eddy Prandtl number was assumed constant.

The nonlinear evolution is illustrated in the perturbation streamfunction and total density fields at 0.8, 1.2, 1.6, and 1.8 wave periods in Figures 11 and 12. The early evolution is clearly dominated by the KH modes as they grow from small initial disturbances. After 0.8 wave periods, however, the KH modes are sufficiently energetic that nonlinearity plays a major role. This is evident in Figure 11 b to d, where a large-scale propagating gravity wave can be seen emerging from the unstable shear layer. The corresponding evolution of the density field shows a large-amplitude gravity wave forming even as the small-scale disturbances in the shear layer decay (Figure 12 b to d).

The gravity wave structure that emerges bears a close resemblance to that illustrated in Figure 3. Evidently, the nonlinear interaction of two KH modes excited a mode B1-like disturbance. Whether this is a consequence of the instability of the initial profile to a B1 mode at this minimum mean Richardson number and horizontal wavenumber is not yet known. However, in view of the strong nonlinear forcing, it seems more likely that the character of the gravity wave is determined by the KH instabilities themselves. Because the most rapidly growing KH modes typically have horizontal phase velocities near zero in a mean profile given by (1), their nonlinear interaction must necessarily excite propagating gravity waves also with horizontal phase velocities near zero. Since B type modes are more likely to have horizontal phase velocities near zero (Lalas and Einaudi, 1976; Fritts, 1980), this excitation mechanism may preferentially select waveforms resembling B type rather than A type modes.

The vertical velocity perturbation and minus the Reynolds stress of the forced gravity wave are illustrated in Figures 13 and 14, respectively. As observed in Figure 11, the gravity wave grows very rapidly through the wave-wave interaction of the two growing KH modes. During this rapid growth, the forced gravity wave, like the KH modes exciting it, is largely confined to the shear layer. This is a reflection not of the modal structure, but of the rapid excitation and relatively slow vertical propagation of the emerging gravity wave. As the KH modes reach large amplitude, however, their growth rates slow, and they begin to dissipate. Likewise, the forced gravity wave ceases to grow and achieves a more global structure. This is evident from both the vertical velocity distribution (Figure 13 c and d), which becomes more structured with time, and the vertical spread of the Reynolds stress, $-\overline{uw}$, from the shear layer (Figure 14 b to d), indicating vertical wave propagation.

The combined vertical Reynolds stresses of the forced gravity wave and the two KH modes act to modify the mean velocity profile according to equation (16). Up to 0.8 wave periods, KH growth is rapid and mean flow modifications are small and confined near the unstable shear (see Figure 15a). As KH growth slows, however, additional shear stabilization results, and evidence of the forced gravity wave appears (Figure 15 b to d). Eulerian mean flow modifications near the unstable shear become permanent as the KH modes are dissipated, and those modifications associated with the outward propagating gravity wave become permanent when and where that wave is dissipated.

As evidenced by Figures 6 and 15, the Eulerian mean flow modifications that result from a nonlinear gravity wave excitation may be substantially smaller than those expected with a quasi-linear excitation. There are two reasons for this. First, the KH modes, in addition to the vertically propagating gravity wave, contribute to the stabilization of the unstable shear. This permits less of the unstable shear energy to become associated with a wave motion that induces mean flow modifications well away from the shear layer (and is, therefore, less efficient at stabilizing the shear layer itself). Second, nonlinear excitation causes the resulting gravity wave to be much more transient than quasi-linear excitation. Thus, even though gravity waves of large amplitude may be excited through the nonlinear interaction of two KH modes, they have less time to alter the mean flow than if they were excited and evolved in isolation.

Perhaps the most significant result of this study is the demonstration that nonlinear excitation enables a large amplitude gravity wave to be excited on a time scale comparable with that required for KH growth. This finding is illustrated in Figure 16, which shows the kinetic energy as a function of time for the radiating

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gravity wave and the two KH modes. For reference, the linear viscous and inviscid growth rates of mode B1 discussed in Section 4.1 are illustrated as well. Up until ~1.0 wave periods, the KH modes grow at very nearly their linear (viscous) growth rates. The radiating gravity wave grows from infinitesimal amplitude at approximately the sum of the KH growth rates until it achieves an energy comparable with that of the KH modes. At t is point, the KH modes begin to be dissipated by the increased eddy viscosity and diffusivity (simulating nonlinear breakdown and turbulent dissipation), but they continue to transfer energy (or action) into the radiating gravity wave. After ~1.4 wave periods, however, the KH modes are unable to interact strongly, and radiating wave energy ceases to grow via nonlinear excitation. The slight increase in radiating wave energy after 1.4 wave periods is a consequence of wave propagation into a region of increasing $(\bar{u}-c)$. In an inviscid fluid, the wave action defined approximately as $A = E/\alpha(\bar{u}-c)$, and not the wave energy, E, is a conserved quantity (Bretherton and Garrett, 1968; Andrews and McIntyre, 1978).

6. SUMMARY AND DISCUSSION

We have presented the results of several numerical simulations intended to illustrate the role of nonlinearity in the shear excitation of atmospheric gravity waves. This study was motivated by the inability of linear theory to account for the observed importance of such waves.

Quasi-linear simulations of the evolutions, in isolation, of modes B0 and B1 were used to examine the effects of wave-induced mean flow accelerations on wave growth and decay. It was observed that modes that are propagating both above and below the unstable shear layer grow by extracting energy from, but ultimately transport mean momentum through, the shear layer. These modes are thus able to effect Eulerian mean flow accelerations well above and below the source shear. It also appears possible that such modes, growing in isolation, can achieve a state of quasi-sustained radiation in which the wave motion initially excited at the shear layer ceases its growth and its modification of the mean velocity shear at the level of excitation, but continues to propagate (perhaps dissipating slowly) and to effect mean flow accelerations elsewhere in the medium.

In contrast to modes that are propagating above and below the unstable shear, mode B0 was found to grow by extracting energy from the unstable shear layer, to stabilize the shear by transporting mean momentum into the shear layer from above, and to undergo a rapid decay as the shear layer became stabilized. Thus, unstable modes that are vertically evanescent below (or above) the shear layer are apparently unable to achieve a state of quasi-sustained radiation. Nevertheless, such modes can propagate and induce mean flow accelerations well away from the source shear and may, therefore, play a role in a variety of other atmospheric processes.

A nonlinear simulation was performed to investigate the excitation of a vertically propagating gravity wave via the interaction of two KH modes with similar wavelengths. It was found that the excitation can be very efficient, resulting in the rapid formation of a transient gravity wave of large amplitude. The transient nature of the resulting gravity wave is a consequence of the rapid growth and subsequent decay of the forcing KH modes. In simulations not discussed, a more gradual excitation at a larger minimum mean Richardson number was found to generate less transient but smaller amplitude radiating waves.

The appearance of a mode B1-like disturbance in the nonlinear simulation is not fully understood at present. It is possible that mode B1 emerges as a consequence of the near-resonant interaction with the two KH modes, as suggested by Chimonas and Grant (1981). But mode B1, in isolation, is slowly growing and emerges because of wave reflection at the lower boundary. The form of the rapidly growing radiating gravity wave excited in the nonlinear simulation, however, cannot be sensitive to the location of the lower boundary because, like the KH modes, it is highly localized near the shear layer during excitation. Only after excitation has ceased does it achieve a more global structure. Thus, it seems more likely that the radiating gravity wave has a mode B1-like structure (rather than a B0, B2, or type A mode structure) because its horizontal phase velocity, as determined by the nonlinear excitation, is most like that of mode B1. This hypothesis is the subject of further study.

The rapid nonlinear excitation of a vertically propagating gravity wave appears to explain the common occurrance of shear-excited gravity waves in the atmosphere. Such an excitation depends crucially on the relative amplitudes of the two KH modes, however. In the simulation discussed in section 5, those amplitudes were adjusted to provide near-optimal radiating wave excitation. Nevertheless, there are several

reasons why we might expect efficient radiating wave excitation in the atmosphere as well. First, a localized perturbation at an unstable shear layer can be expected to produce a localized (broad-banded) rather than a horizontally uniform (monochromatic) response. Second, nonlinearity may act to distribute energy throughout a range of wavenumbers. And perhaps most important, unstable shear layers in the atmosphere are surely not horizontally homogeneous. Thus, the shear layer itself may act to select the dominant radiating wave scales by confining KH growth to the (most) unstable regions.

Analytical and numerical studies have contributed much to our understanding of the shear excitation of atmospheric gravity waves. However, there remains much that is not known. One method that holds much promise for the advancement of our understanding of this and other excitation mechanisms is the direct observation of atmospheric processes. Of particular utility in this capacity are the various operational and planned atmospheric radars. These and other measurement tools can provide a wealth of data on gravity wave excitation, propagation, and dissipation, and their use for this purpose is strongly encouraged.

ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

Figure 1.   Unstable mode boundaries for $z_i/h = 20$ and $\sigma = 0.01$. Type A and B modes are illustrated in (a) and (b), respectively. Dashed lines in (b) indicate overlap with type A modes.

Figure 2.   The real (———) and imaginary (----) vertical velocity perturbations for each unstable mode for $z_i/h = 20$, $\sigma = 0.01$ and $Ri = 0.10$. The corresponding horizontal wavenumbers are 0.56, 0.32, 0.28 and 0.25 for the A0, 1, 2, and 3 modes and 0.31, 0.18 and 0.06 for the B0, 1 and 2 modes, respectively. Units are arbitrary.

Figure 3.   Perturbation streamfunction (a) and total density (b) fields for mode B1 at 40 wave periods in the quasi-linear model. Parameters are those in Section 4.1.

Figure 4.   Horizontal (a) and vertical (b) perturbation velocities for mode B1 at 40 wave periods in the quasi-linear simulation of Section 4.1.

Figure 5.   Minus the Reynolds stress for the quasi-linear mode B1 simulation at (a) 20, (b) 30, (c) 40, and (d) 50 wave periods. Units are MKS.

Figure 6.   Eulerian mean velocity profile for the quasi-linear mode B1 simulation at (a) 20, (b) 30, (c) 40, and (d) 50 wave periods. Units are MKS. The dashed line denotes the initial velocity profile.

Figure 7.   Perturbation streamfunction (a) and total density (b) fields for mode B0 at 16 wave periods in the quasi-linear model of Section 4.2.

Figure 8.   Horizontal (a) and vertical (b) perturbation velocities for mode B0 at 16 wave periods in the quasi-linear simulation of Section 4.2.

Figure 9.   Minus the Reynolds stress for the quasi-linear mode B0 simulation at (a) 10, (b) 12, (c) 14, and (d) 16 wave periods. Units are MKS.

Figure 10.   Eulerian mean velocity profile for the quasi-linear mode B0 simulation at (a) 10, (b) 12, (c) 14, and (d) 16 wave periods. Units are MKS.

Figure 11.   Perturbation streamfunction field at (a) 0.8, (b) 1.2, (c) 1.6, and (d) 1.8 wave periods for the nonlinear simulation of Section 5. Note the mode B1-like disturbance emerging from the shear layer.

Figure 12.   Density field at (a) 0.8, (b) 1.2, (c) 1.6, and (d) 1.8 wave periods illustrating the KH growth and radiating wave excitation observed in the nonlinear simulation of Section 5.

Figure 13.   Vertical velocity perturbation of the forced gravity wave at (a) 0.8, (b) 1.2, (c) 1.6, and (d) 1.8 wave periods showing the increase in vertical structure as wave growth slows.

Figure 14.   Minus the Reynolds stress of the forced gravity wave illustrating the propagation of wave energy (or action) away from the source shear. Times are (a) 0.8, (b) 1.2, (c) 1.6, and (d) 1.8 wave periods.

Figure 15.   Eulerian mean velocity profile for the nonlinear simulation of Section 5. Times are (a) 0.8, (b) 1.2, (c) 1.6, and (d) 1.8 wave periods. Units are MKS.

Figure 16.   Kinetic energy of the KH modes of 400 π (– –) and 500 π (- - -) in wavelengths and the radiating gravity wave (———) as a function of time. Energy units are joules/m. The linear viscous and linear inviscid mode B1 growth is illustrated for comparison. Time is in wave periods.
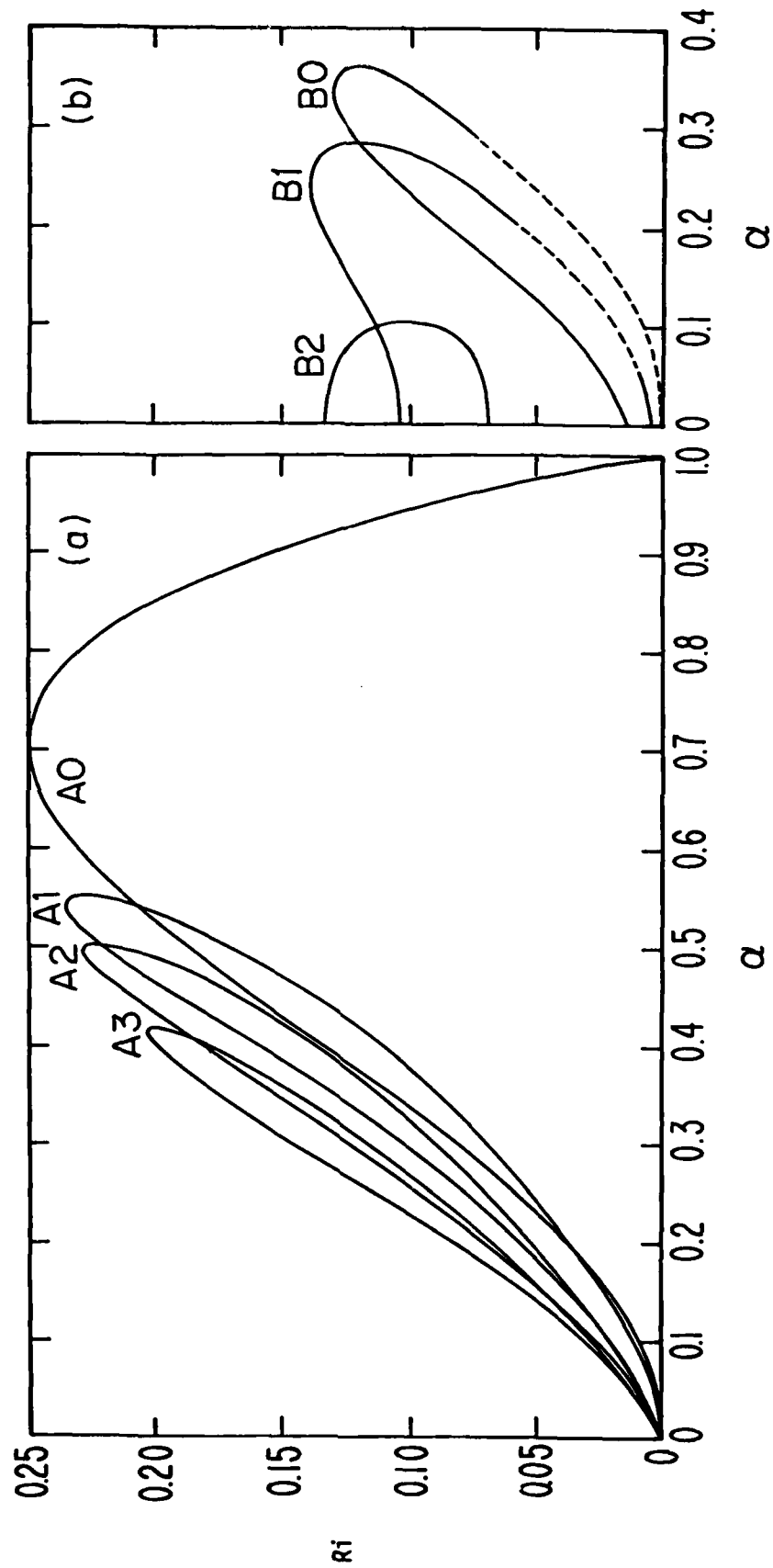
Figure 1

Figure 2

Figure 3

(b)

z_i/h

(a)

z_i/h
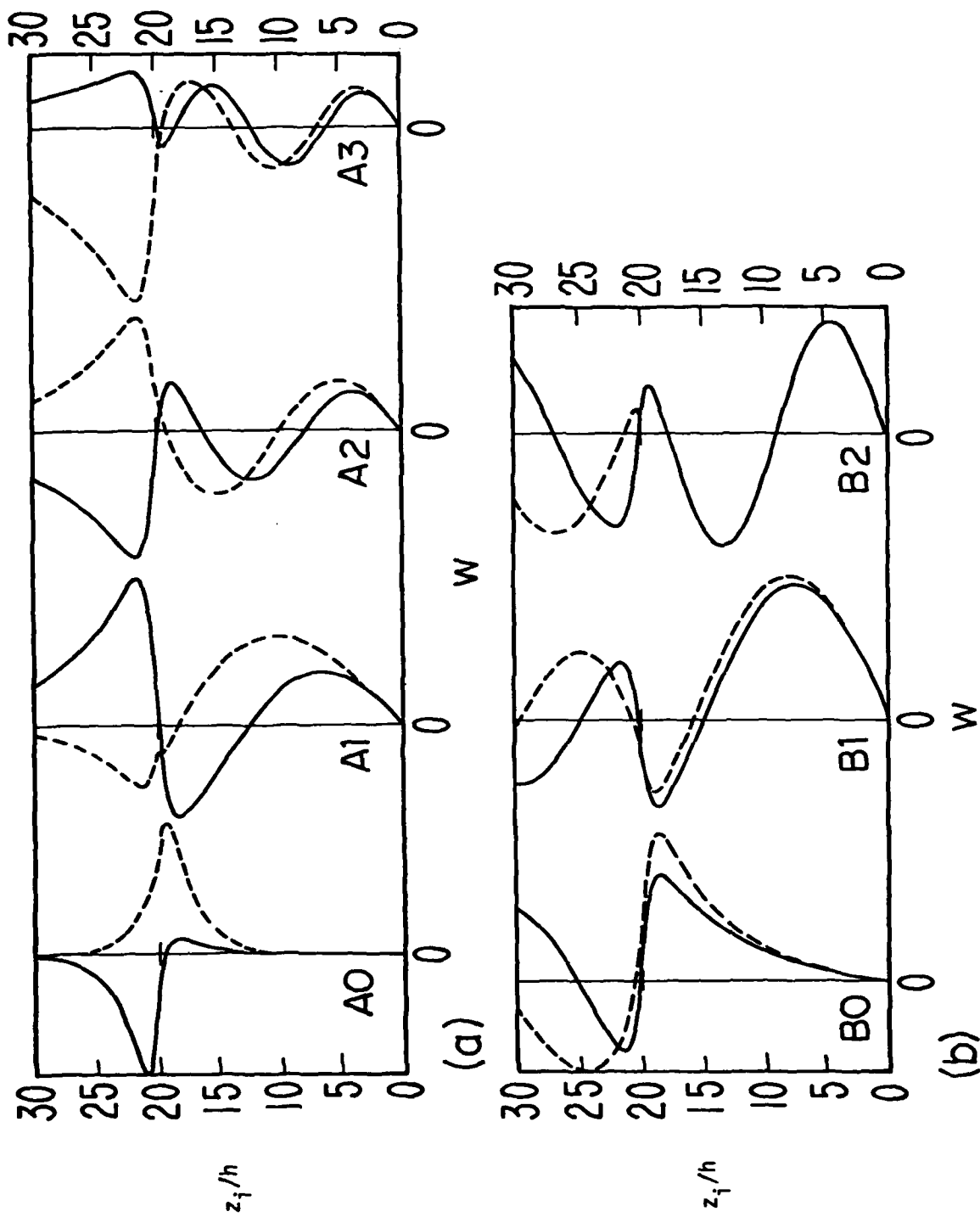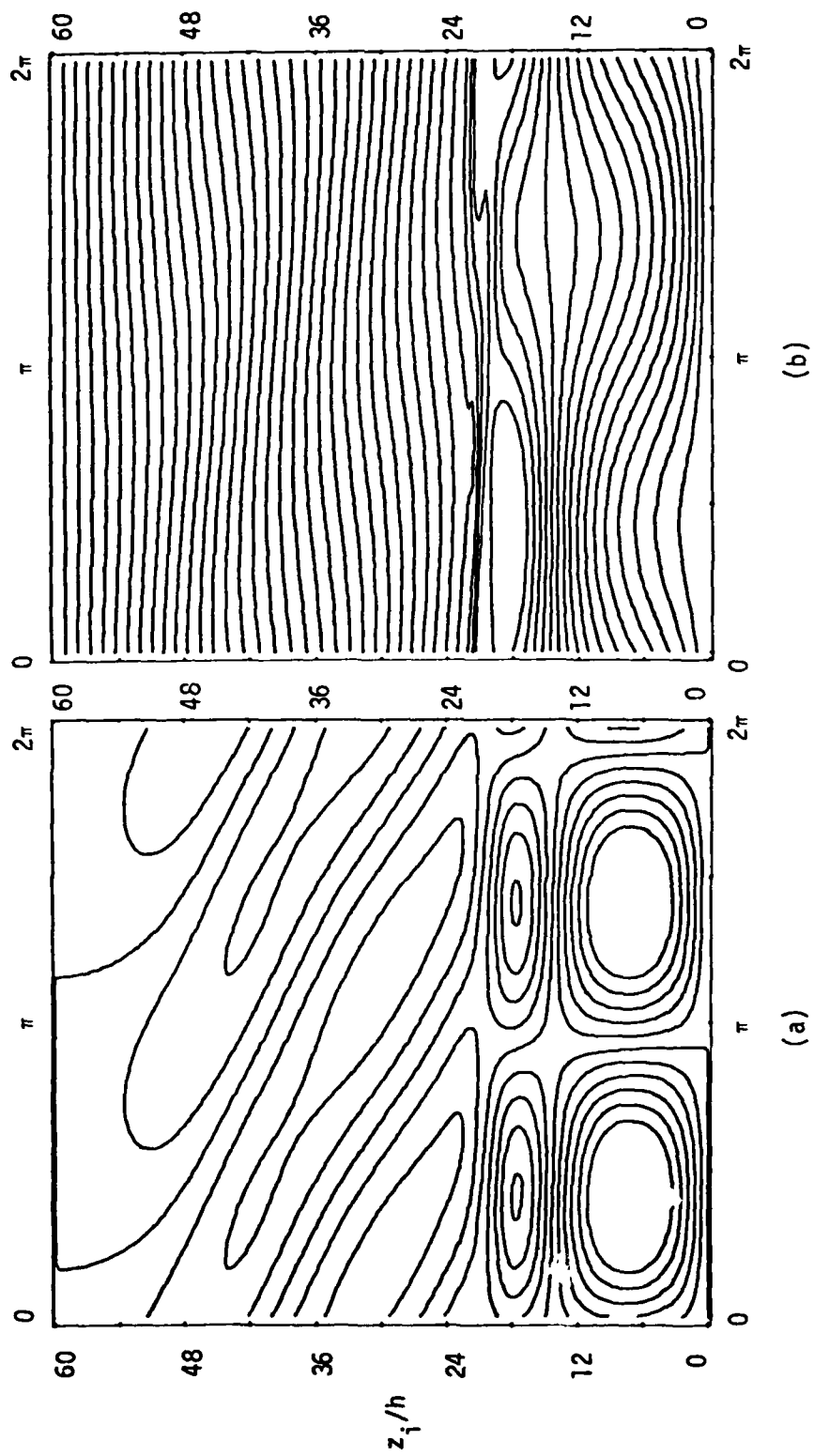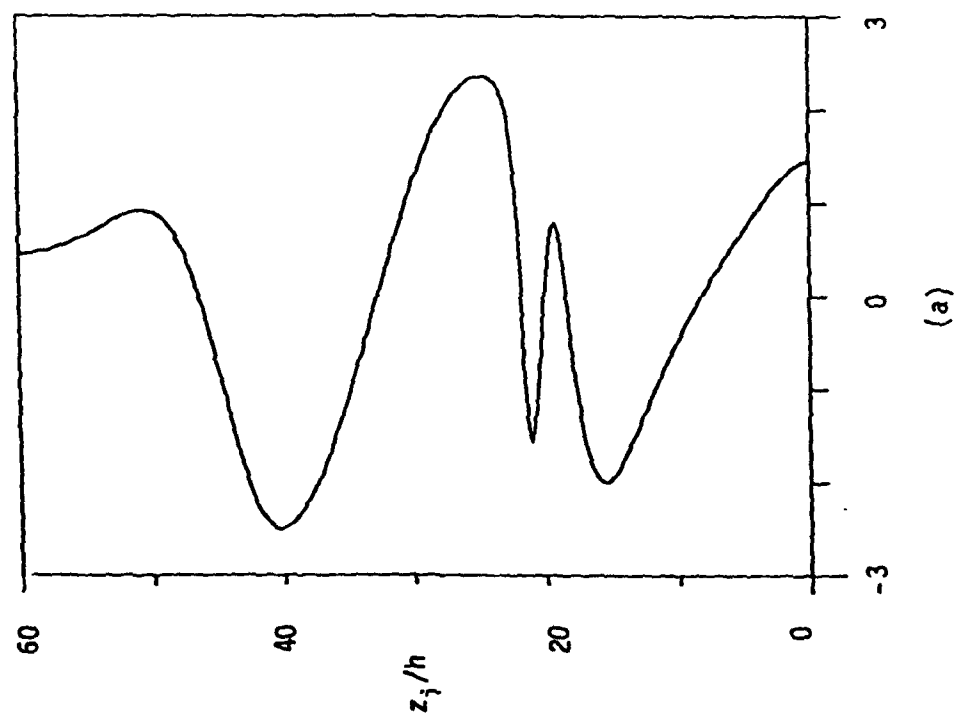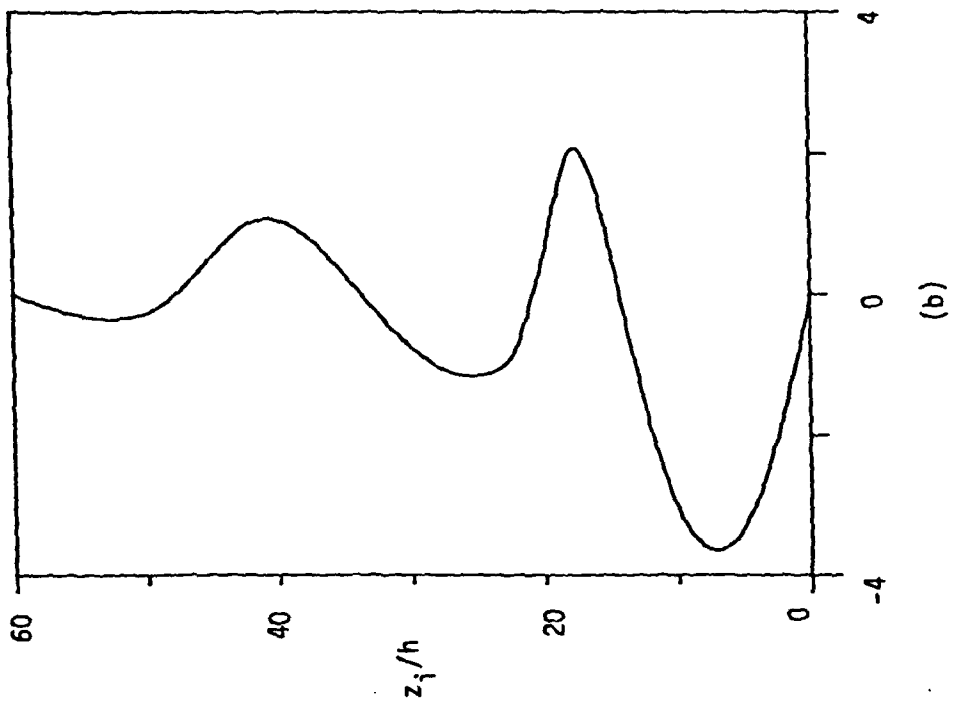
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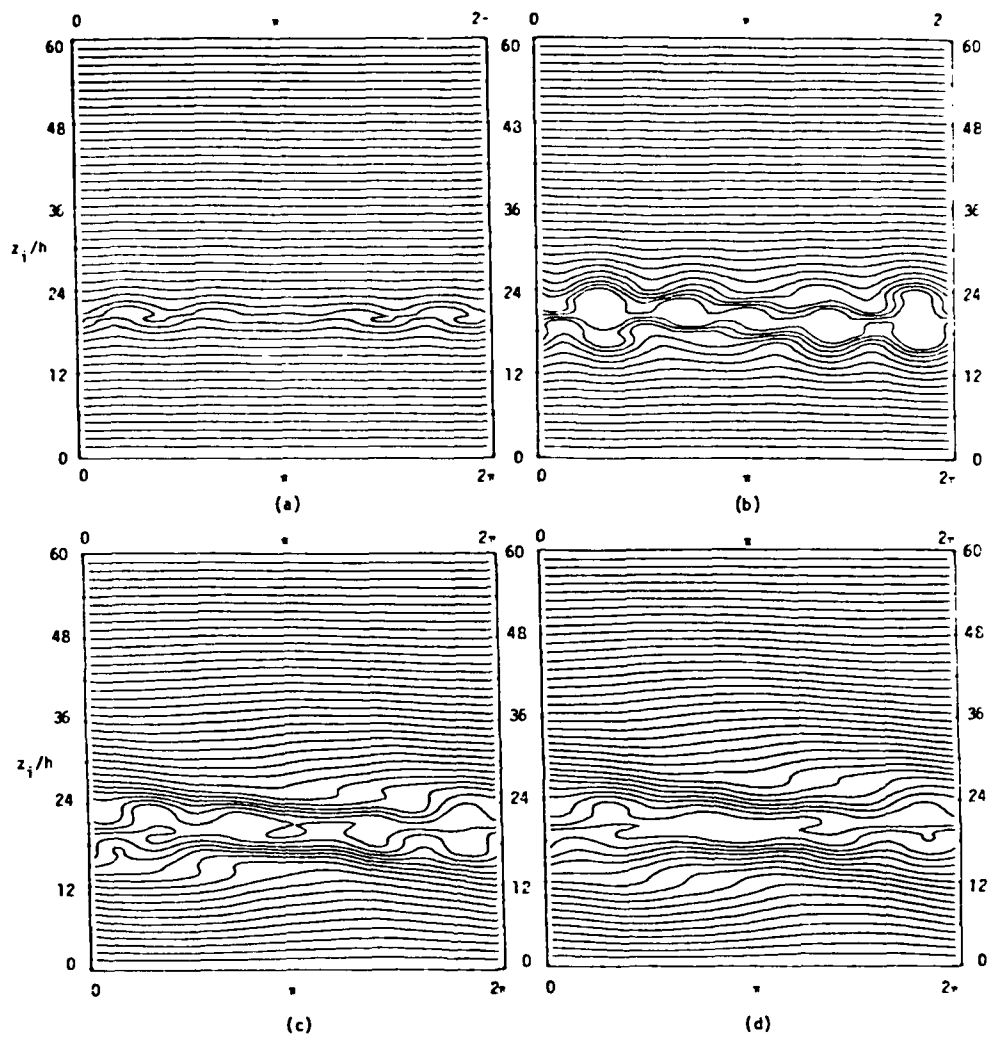
Figure 5

Figure 6

z_i/h
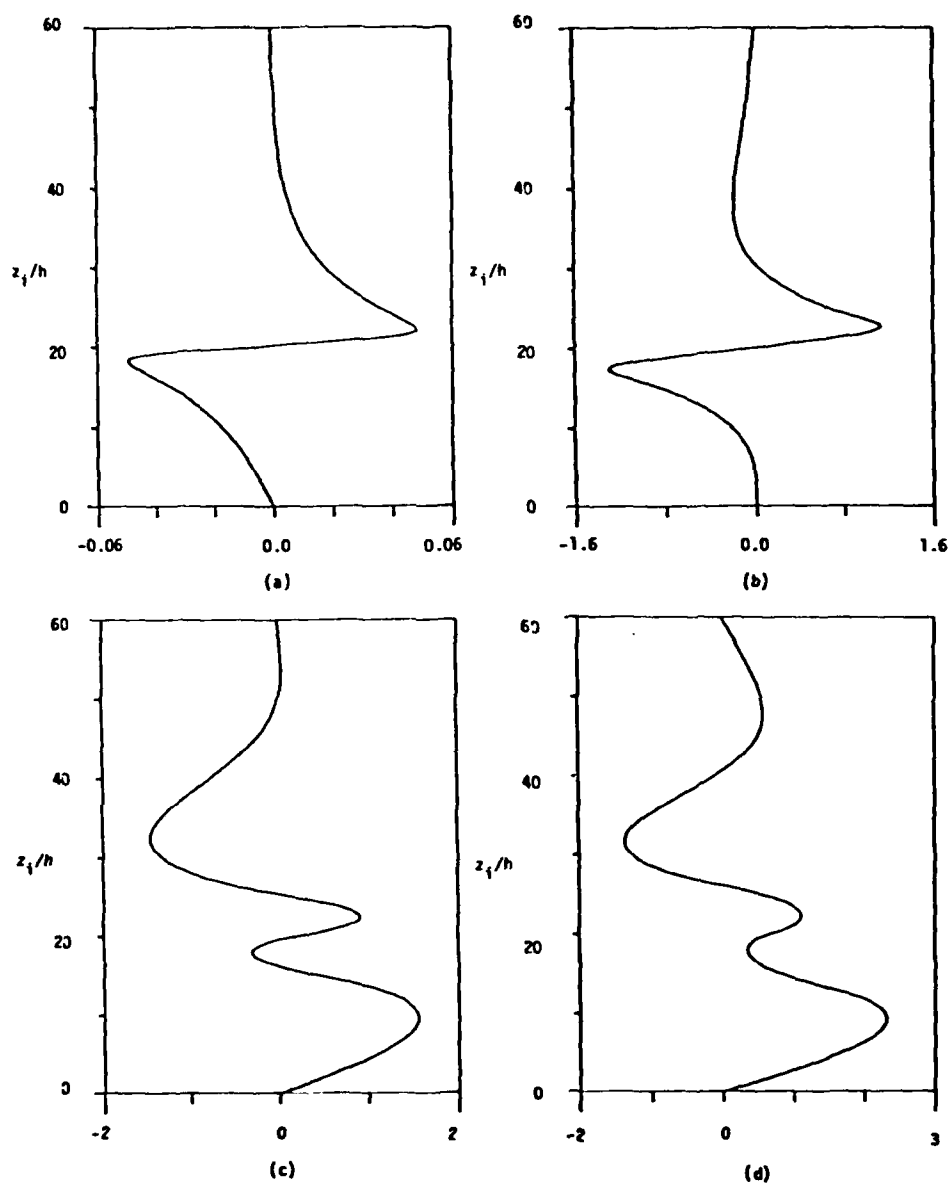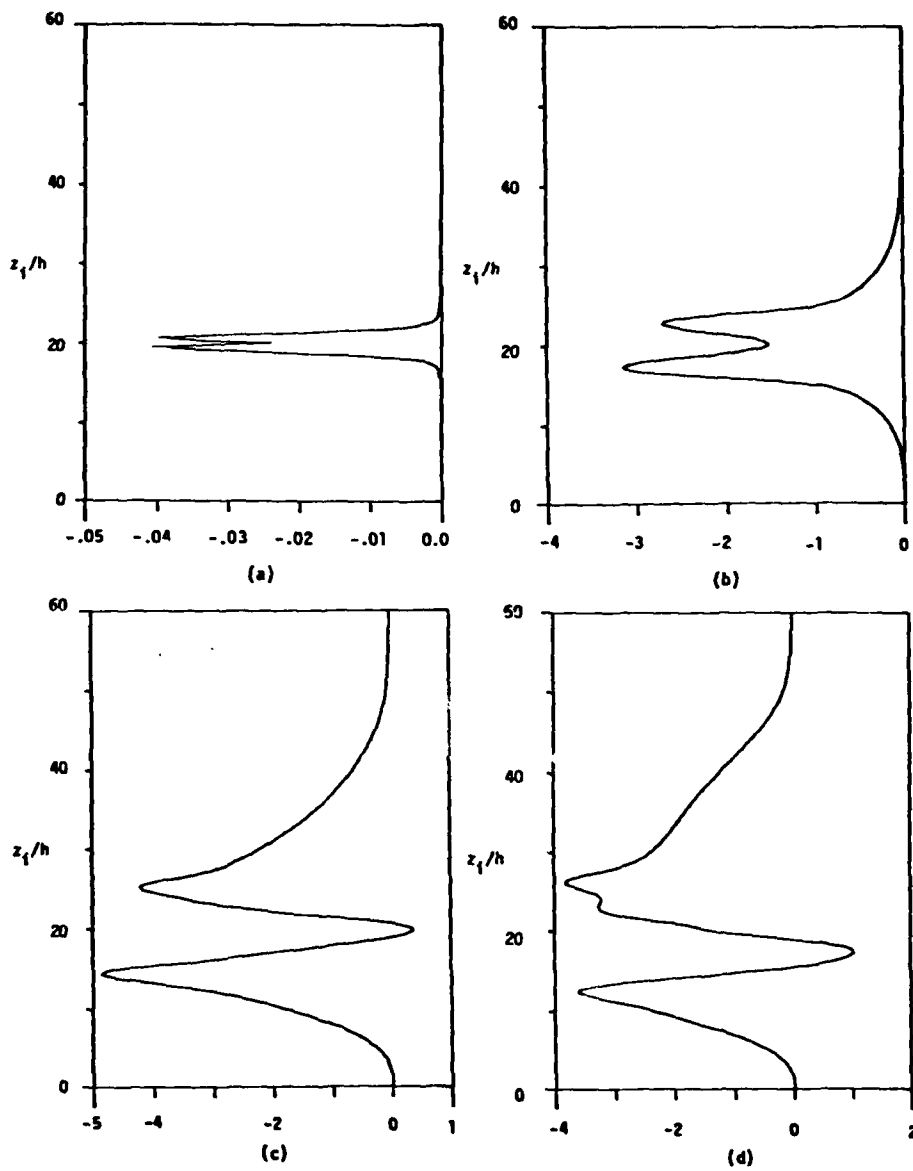
(a)

(b)
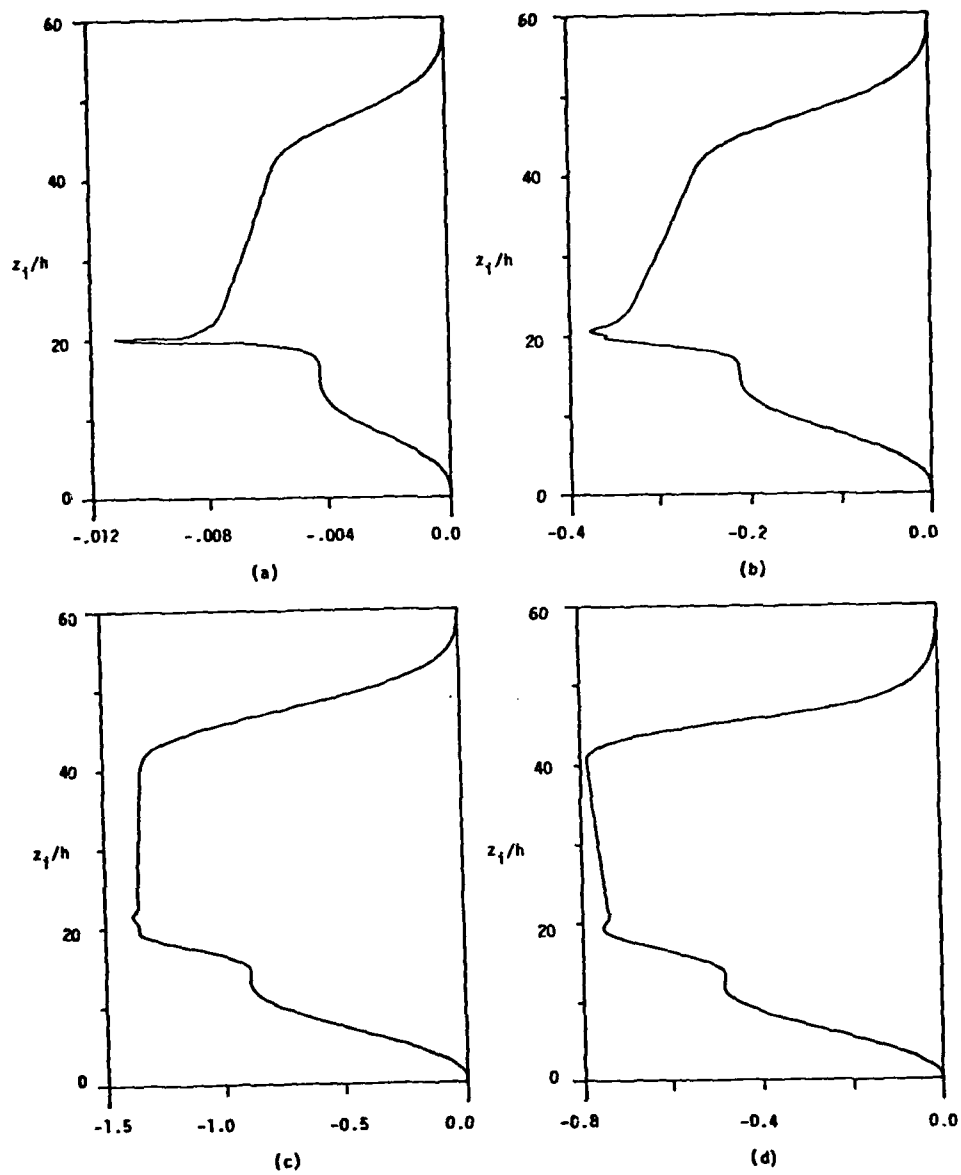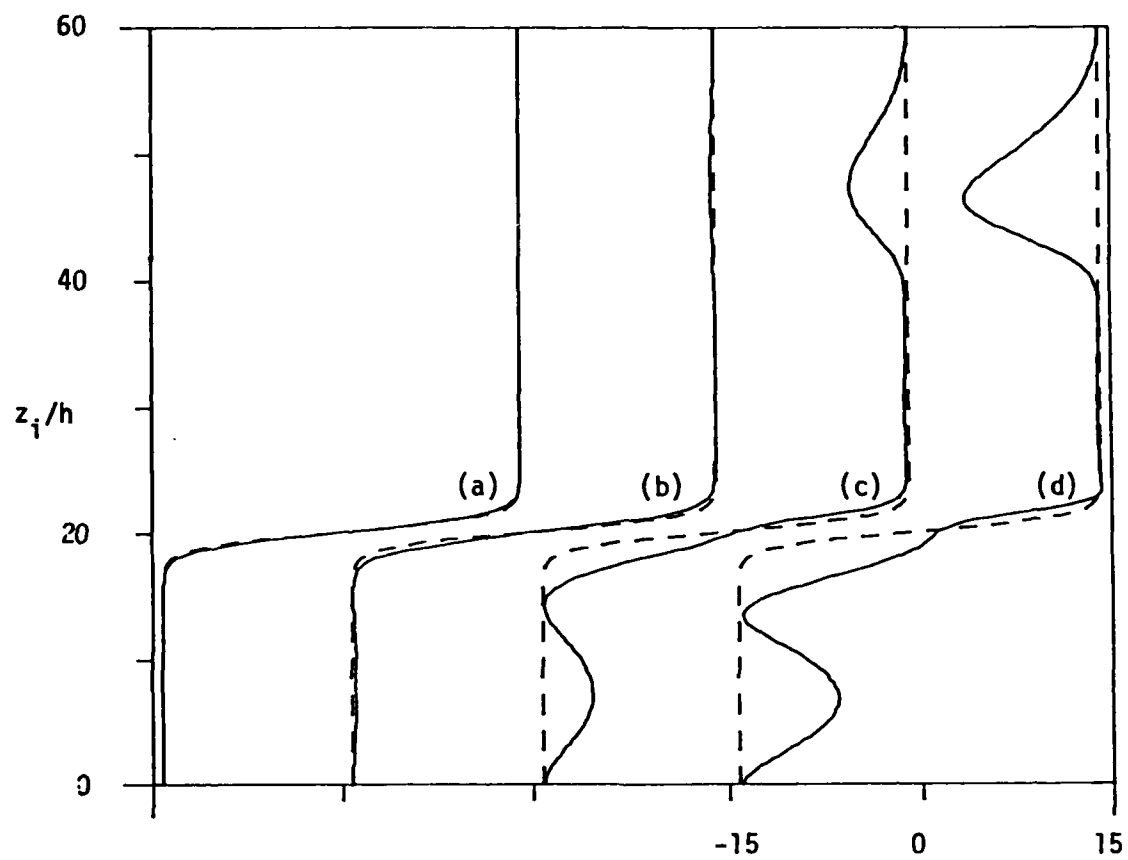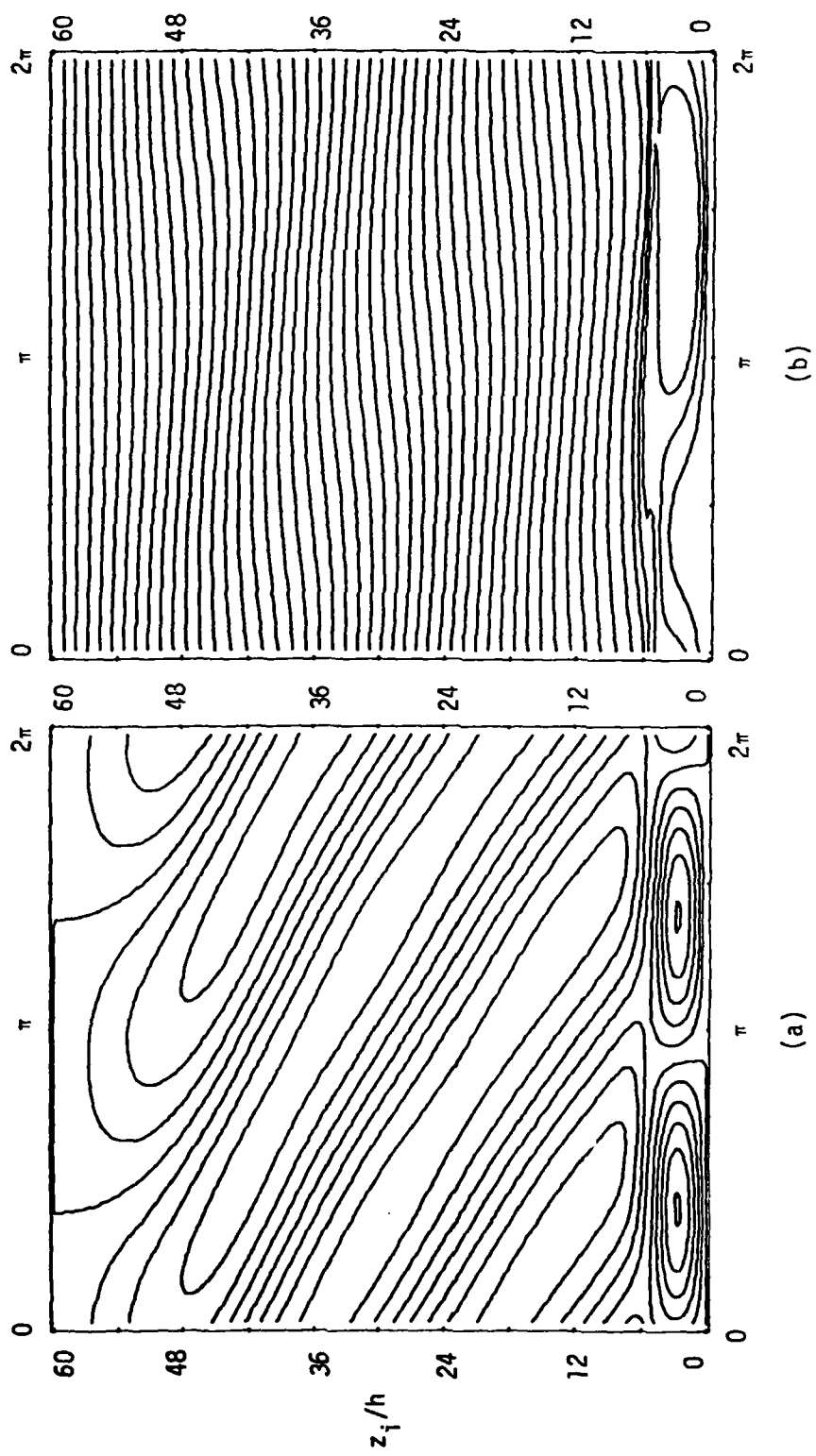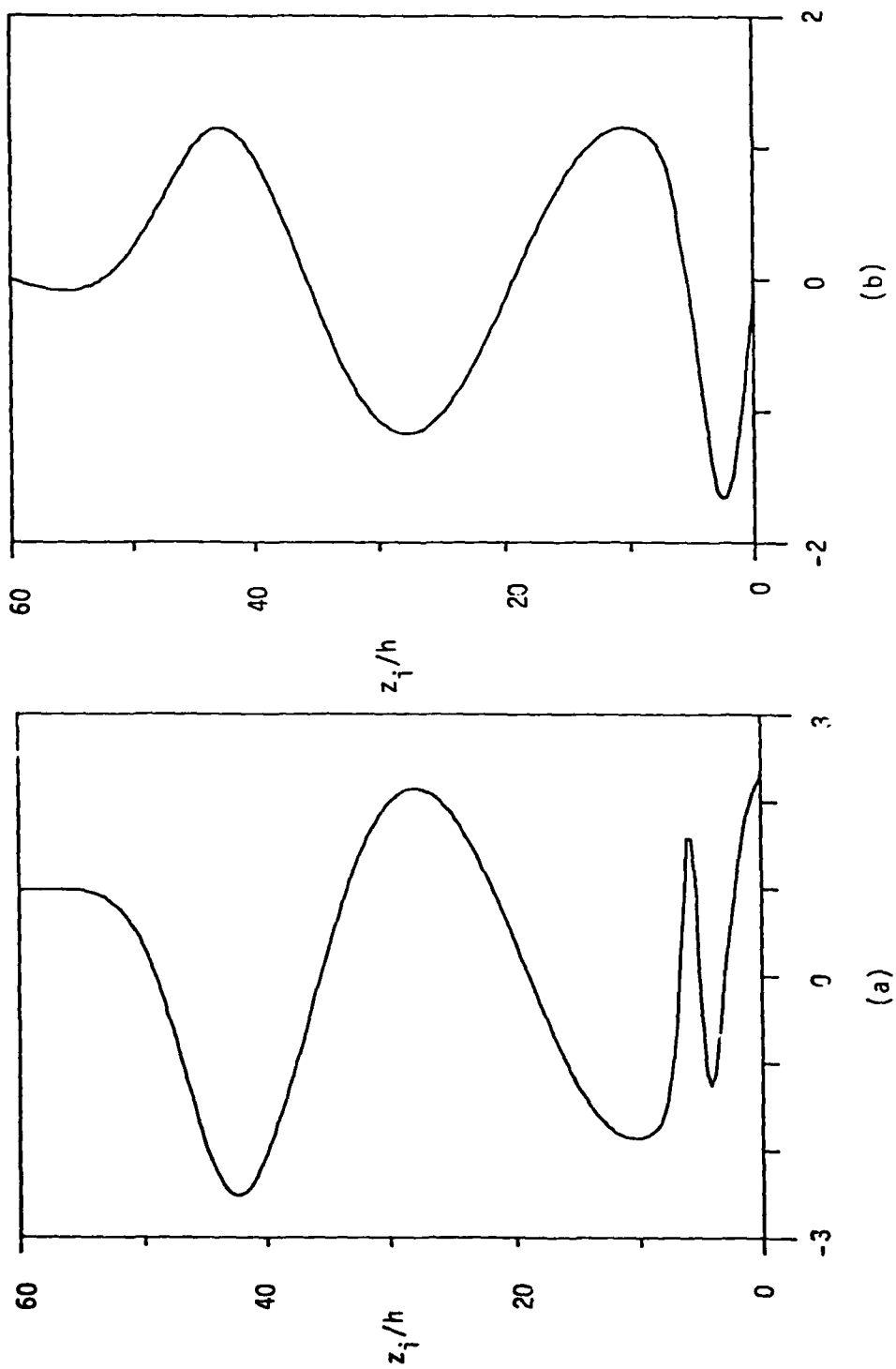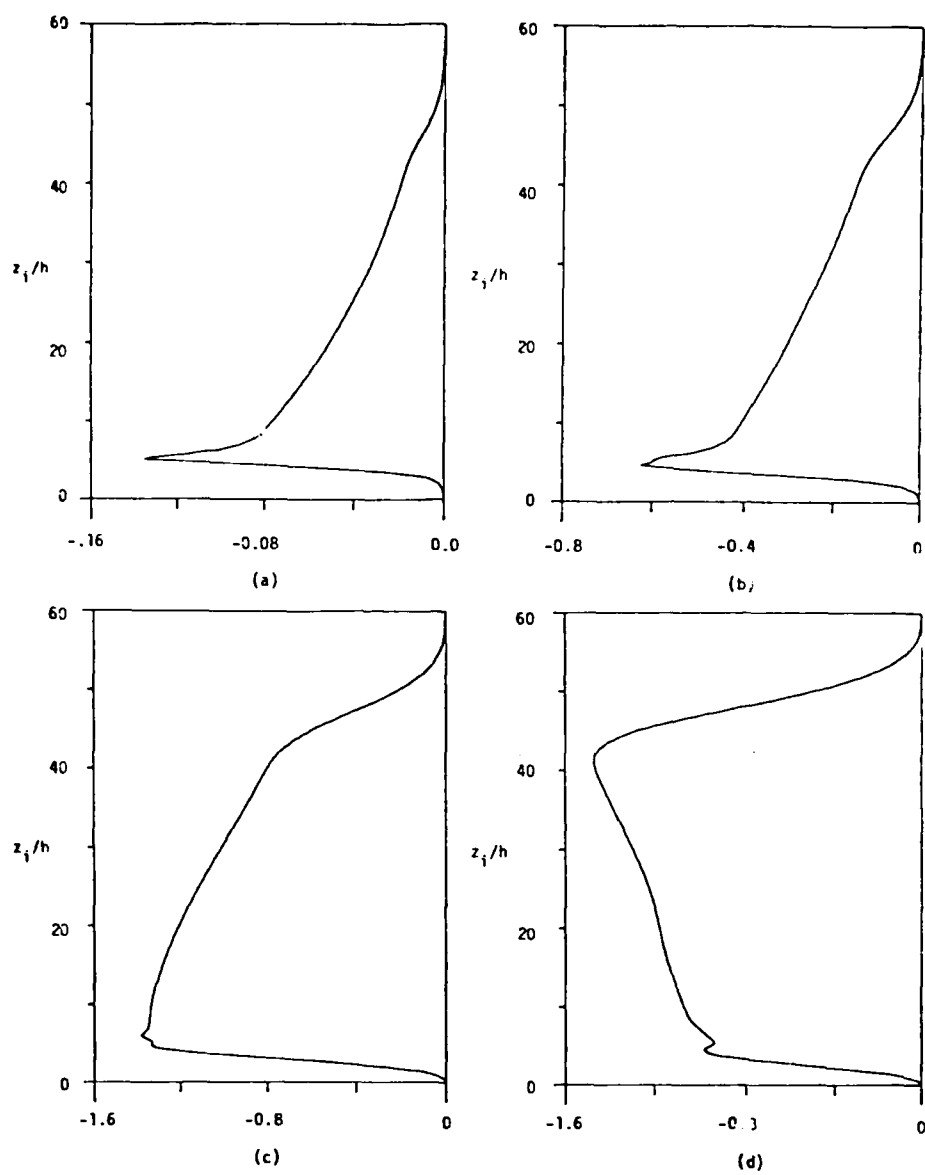
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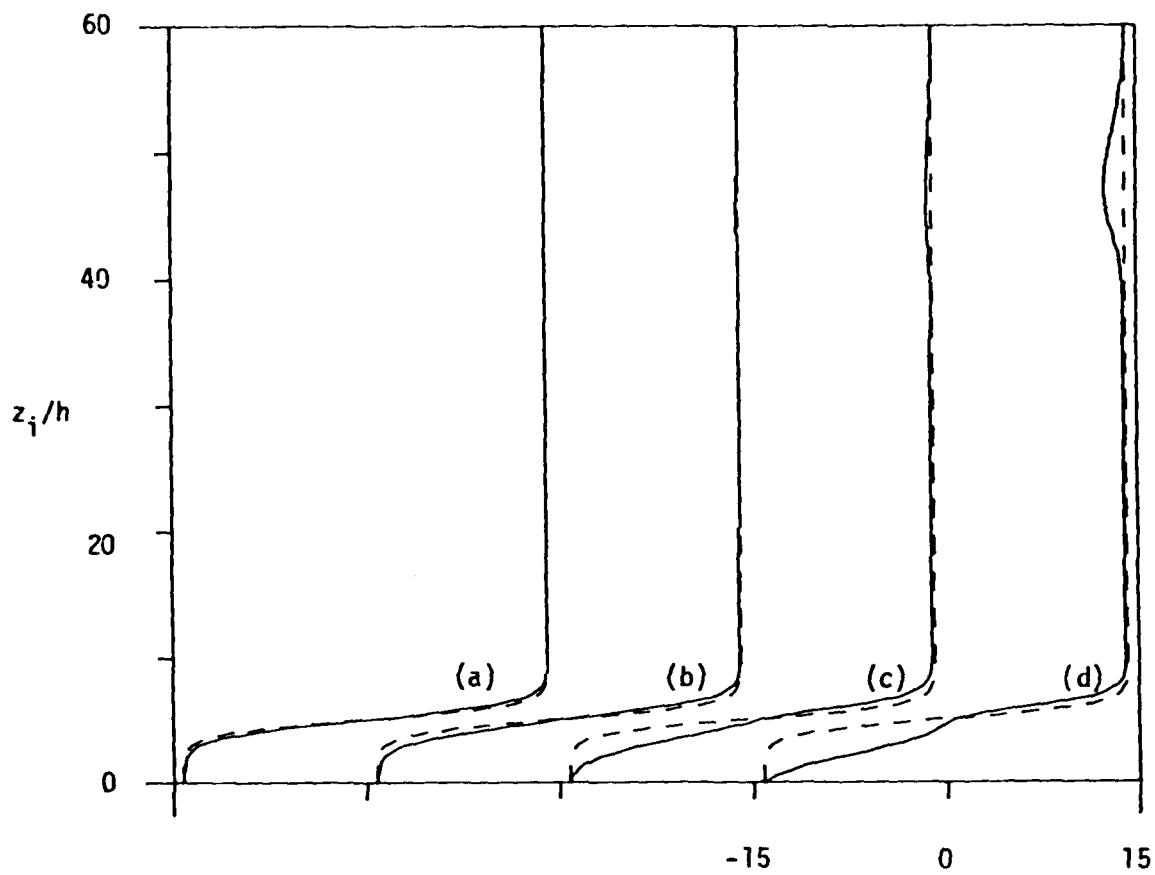
Figure 8
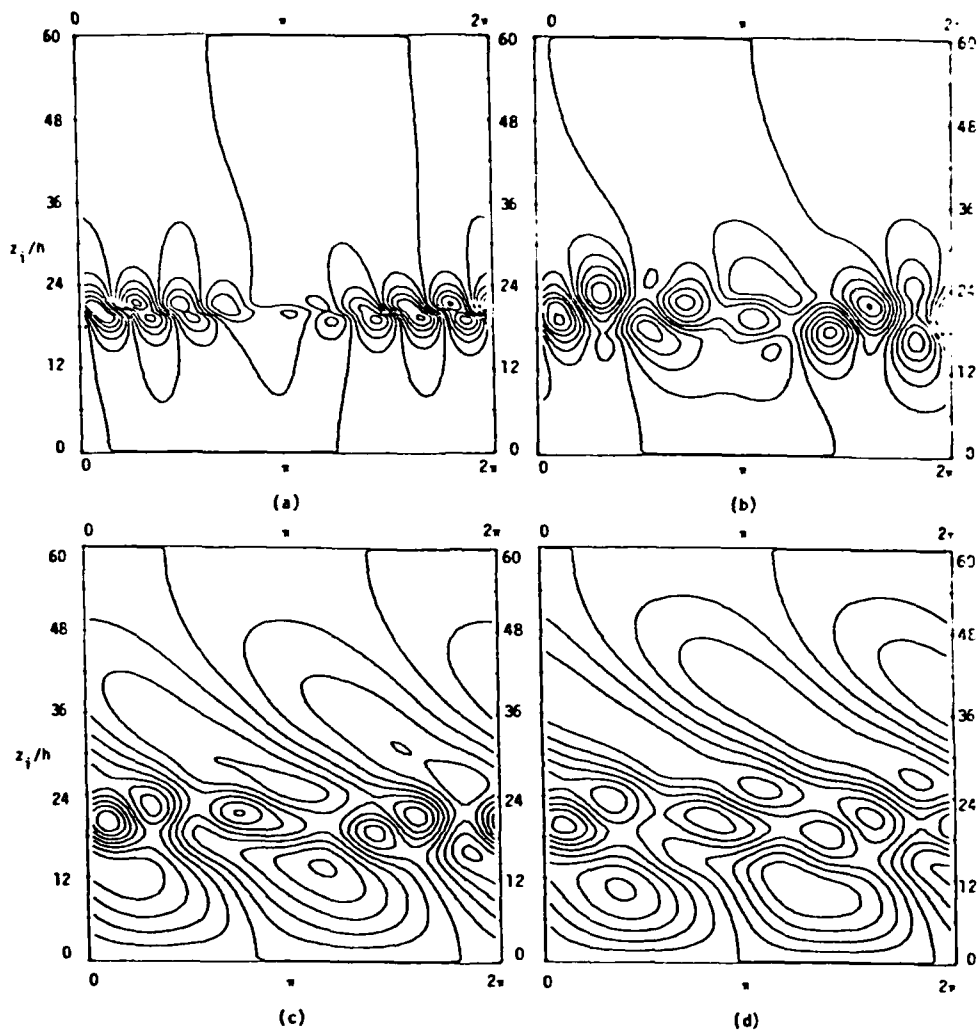
Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16